VARIATIONS OF THE WATER STOCK OF SOIL IN A BARE GROUND PROFILE

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SUMMARY

On the basis of a twenty years (1951 – 70) series of soil moisture measurements made at Erdőhát (Hungary) average and extreme variations of the water stock of soil (w) are dealt with, such as: yearly variation, changes with depth as well as variability of the annual values.

It is sometimes practical to compare the variations of the water stock and certain hydrophysical constants. The average water stock variations show the following characteristic features: the water content decreases with depth throughout the whole year and the moisture profile is not a linear one; the yearly variation is asymmetrical and it consists of a longer period of drying out and of a shorter interval of moistening; a phase shift is experienced with depth, in the yearly average the upper layer of the soil (down to 70 cm) is relatively more saturated, while in the lower layers a relative dryness is experienced; the extremes of the moisture profile at depth develop in June, respectively in December. In a more or less empirical way depth functions of the yearly average soil moisture depth-profile as well as that of the average yearly amplitude have been derived. Matching of the empirical and computed values is very good.

A simple model of extreme water content changes is presented and a lot of new characteristics defined, such as: the absolute yearly oscillation of the water stock (D_{I}), amplitudes of the extreme-functions (d_1, d_2) and their sum (D), the wideness characteristics (d_3, d_4), the distortion index (Δ), the passive, respectively fossil water stock etc. The yearly variation of $w_{\max}(z)$ (maximum water stock as a function of depth) as well as the distribution of the fossil water stock with depth is shown. The empirical depth-functions of the extreme amplitudes (d_1, d_2) have been determined and the depth-function of the sum of the amplitudes is represented by a differential equation. Some other characteristics show similar features as the extreme amplitudes. The relative distortion index (Δ/D_{I}) is in connection with the relative water saturation state of the soil. The ratio of the extreme oscillation to the average water-stock is expressed by the function: $p(z,t)$, the yearly variation of which shows three types. The connection between average and extreme water stocks is represented by means of the expression: $D/\overline{w}$ and $D_{\mathrm{I}}/\overline{w}$. The depth function of the absolute yearly oscillation has also been derived.

Introduction

Our knowledge about the characteristic features and rules of the variations of the water stock of soil is still rather scanty. The water content of the bare soil depends on the weather and on the hydrophysical features of the ground profile. Proved we have a sufficiently long and detailed moisture observation series from a given for the soil profile we are able to separate the role of the weather and soil in the variations of the water stock of the soil. The soil moisture observation series of Erdőhát of twenty years (1951 – 1970) seems to be suitable for such analyses. As a first approximation we restrict ourselves here to a detailed descriptive discussion of variations in time and space of the water stock of the soil. We will study the average and extreme variations of the water content of the soil, especially with regards the drying out and moistening processes. Only studies made with the use of monthly averages of the layers will be shown.

Hydrophysical contants of the soil determine the frames of the possible variations in time and space of the moisture of soil, thus they serve as an important basis for the considerations (Table 1.)

Table 1.

Values of the most important hydrophysical constants (mm) Erdőhát

Layer cm	W'_k	H'_p	$h W'_k$	$mk W'_k$	$t W'_k$
0 – 25	74	27	47	103	106
25 – 50	61	23	38	102	120
0 – 50	135	50	85	205	226
50 – 100	109	40	69	204	242
100 – 150	81	25	56	213	233
150 – 200	75	16	59	209	217
0 – 100	245	90	154	409	468
0 – 200	400	131	269	831	918

In the soil of Erdőhát the following values of the hydrophysical parameters: field water capacity (W_k), wilting point (H_p), and the useful water capacity (hW_k) are diminishing with the depth, while the maximum capillary water capacity (mkW_k) and the total water capacity (tW_k) remain practically constant. So the depth distribution of soil moisture can not be considered a simple function of the weather. It is more practical to make more comparisons – and these in various ways – between variations of the soil moisture and the various hydrophysical parameters.

The climatic factors influencing for the most part the water balance of the soil at Erdőhát are: yearly average of rainfall: 555 mm, in the summer half year: 328 mm, yearly total of evaporation on bare ground: 388 mm, in the summer half year: 281 mm, etc. (E r d ő s, 1975). The observations parcels are surrounded by soil wall, thus a surface runoff is not possible.

1. Average variations of the water stock of the soil

The variations of the average water stock of the soil layers are shown in Table 2. In the course of analyses layers with dm thickness are also used, but they will not be shown owing to their extent. On the basis of the Table some general features can be seen and a few important conclusions can be drawn.

Table 2.

Mean water stocks of various soil layers (mm)
Erdőhát, 1951 – 70

Layer (cm)	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
0 – 25	78	79	71	62	59	57	53	50	51	56	67	74
25 – 50	59	60	60	55	53	50	49	46	46	45	52	57
0 – 50	137	139	131	117	112	107	103	96	96	102	119	130
50 – 100	90	93	97	89	86	83	79	73	73	71	78	86
100 – 150	68	73	76	68	67	67	65	58	58	56	61	62
150 – 200	66	70	65	67	65	61	63	57	56	54	56	59
0 – 100	227	232	228	206	198	190	182	169	169	173	199	216
0 – 200	365	376	369	339	330	317	310	286	284	285	316	337

The water stocks of the layers are diminishing with depth throughout the whole year, except for the 0 – 10 cm layer, in which (and only in this) the layering of moisture in summer is reversed, but this does not disturb the order of distribution of the average values of the 0 – 25, cm respectively the 0 – 50 cm layers any more. At the same time we see that the depth profile of the moisture content is not linear.

For the average yearly variation of the water stock of the soil is characteristic a typical maximum at the and of the winter, as well as a minimum at the end of the summer, respectively in autumn. Thus, the yearly variation divides into a longer drying-out section and a shorter moistening period. We can abserve a phase shift of the extremes with increasing depth. Based on a more detailed data (dm layers) it can be shown that the maximum occurs in February in the the upper layers, and it oscillates between February and March: in the deeper layers the minimum in the layer between 0 and 30 cm falls to August, between 30 and 50 cm to September, between 50 – 180 cm to October and between 180 and 200 cm to November. On more hard grounds, e. g. at Karcag the minimum of soil moisture will be shifted to December already at a depth of 100 cm. (E r d ő s – M o r v a y, 1961). Since the phase shift of the minimums with depth is greater than that of the maximums, the ratio of drying-out and moistening intervals is also shifting with depth: the moistening section becomes shorter and the drying-out interval seems to lengthen with depth.

In reality, also the intensive drying out section becomes shorter at greater depth, because in the first half of the year a rather long equilibrium interval of nearly constant moisture is inserted. Thus in the autumn in the whole layer (0 – 200 cm) drying out and moistening processes take place simultaneously, and in the upper layers the moistening action is predominating already, when in the lower parts the drying out is still very active.

The winter end saturation of the profile of the soil reaches the free ground water capacity only in the 0–50 cm layer, while in the entire layer it only approximates it. On the other hand, during drying out the water stock does not decrease in either layer down to the Hp (except for the layer at 0–10 cm depth), i. e. the bare soil stores a significant amount of utilizable water during the whole year.

The average yearly water stocks of the soil layers can be represented by a depth function independent of the time, this function is denoted by $\overline{w}(z)$. We have seen that the depth distribution of the moisture of the soil is always decreasing and non-linear (Table 2.). This is valid of course for the $\overline{w}(z)$ function too. The relative depth distribution of the yearly average soil moisture can be characterized in connection with the water capacity.

The depth function of the relative differences of the two variables shows a very interesting connection (Fig. 1.) In a yearly average the relative water saturation of the soil is increasing down to a certain depth, then it starts decreasing. The whole layer can be separated into a relatively more saturated zone extending down to 70–80 cm (where the relative differences are below 20%), and a relatively less saturated one (downwards from 70 cm), where the relative differences surpass 20% and the "saturation deficit" increases with depth. The relative saturation of the soil is highest around 40–50 cm and lowest at the greatest depth (around 200 cm).

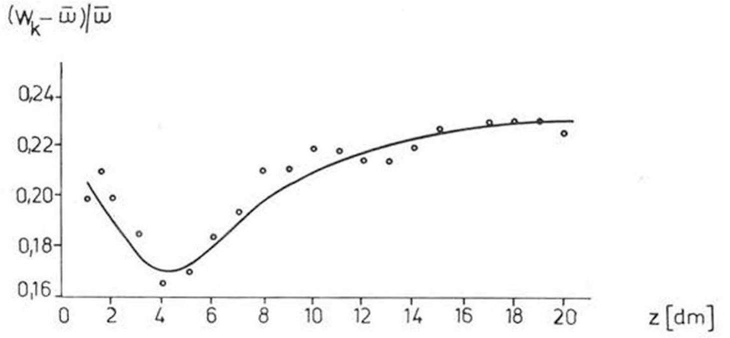

Fig. 1. Yearly mean relative water saturation of the soil, Erdőhát, 1951–70.

Of course, this relative saturation follows different depth distributions according to the seasons. E.g. in the winter half year the depth function of the relative differences is increasing monotonously in the whole layer, while in the summer half year it is very definitely decreasing down to 60 cm and it remains constant at greater depth. Considering these facts we could explain the curve of Fig. 1. in a somewhat paradoxical manner as follows it represents the summer type on its upper section (where the saturation deficit is smaller) and the winter type appears on its lower part (where the saturation deficit turns to be greater).

The mean yearly oscillation ($\overline{w}_{\max}-\overline{w}_{\min}$) of the water stock of the soil is decreasing with depth. If we take a series of doubled (and intertwined) soil layers, e. g. layers of depth limits such as: 0–25, 0–50, 0–100, 0–200 cm, then the average values of the yearly oscillation (amplitudes) become one after another: 29, 43, 63 and 92 mm (Table 2.). We can detect a regularity on sight, if the thickness of the soil layer to be considered is doubled, the average yearly oscillation of the water stock within the layer increases only by factor of 1.5.

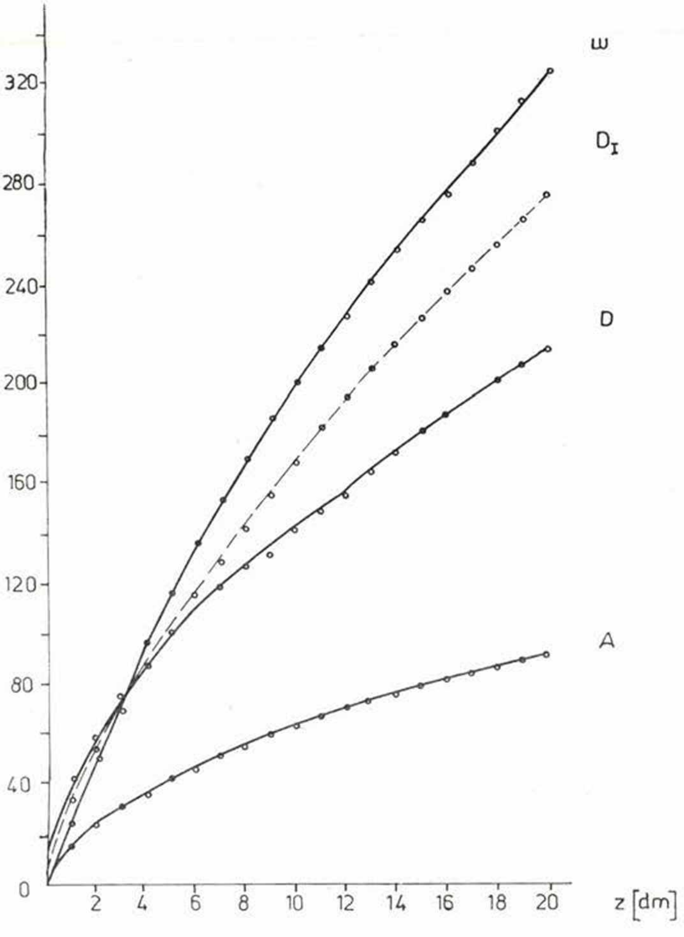

Fig. 2. Empirical cumulative depth- curves of important water stock characteristics Erdőhát, 1951–1970.
D_I – absolute yearly oscillation, D – sum of the extreme amplitudes, A – mean yearly amplitude, w – yearly mean water stock

By summing data of a fine 10 cm subdivision we can easily construct the empirical cumulative curve of the average yearly amplitude, which can be seen in Fig. 2.

Strating from the empirical connection above we have derived the depth function of the average amplitude total (E r d ő s, 1975) in the following form:

$$A(z) = A_1 \frac{B^{\lg z}}{z}, \tag{1}$$

where A_1 denotes the average amplitude in a starting layer chosen arbitrarily, B is a general constant, z is a linear dimensionless depth variable. The true amplitude – depth function – by means of which the amplitudes can be computed at any depth – can be obtained by differentiating equation (1):

$$A'(z) = A_1 B_1 \frac{B^{\lg z}}{z^2}, \tag{2}$$

where $B_1 = \ln B \lg e - 1$. The curve computed from equation (2) together with the empirical amplitude values is shown in Fig. 3. Computed and observed amplitude values fit very closely (except for a few cases).

From the empirical amplitude and from the data of the yearly average water stock we have constructed an empirical quotient-depth –

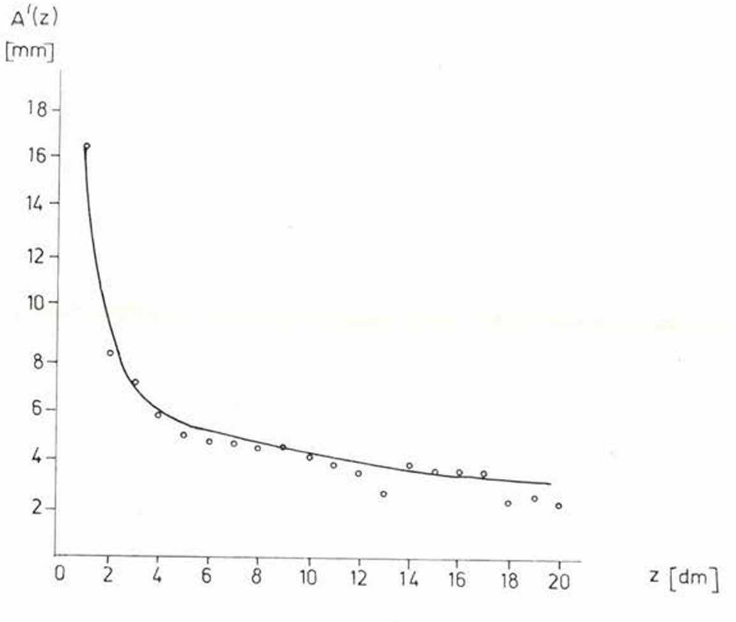

Fig. 3. The curve of the real amplitude-depth function together with the empirical values. Erdőhát, 1951–70.

function. Due to the linear transformation of the curve this function can be only a hyperbola, as follows:

$$\frac{A(z)}{\overline{w}(z)} = a + \frac{b}{z}. \tag{3}$$

It is justified to suppose that the depth function of the average water stocks is in a close and simple connection with that of the average yearly amplitude. From (1) and (3) we can obtain the depth function of the yearly average waterstock-total (Fig. 2.), as follows:

$$\overline{w}(z) = A_1 \frac{B^{\lg z}}{az+b}, \tag{4}$$

where a and b are empirical constants obtained by the use of least squares method. By derivation of the equation (4) we get the depth-profile function of the yearly average soil moisture in the following form:

$$\overline{w}'(z) = A_1 \frac{B^{\lg z}}{az+b}\left(\frac{M}{z} - \frac{1}{z+m}\right), \tag{5}$$

where: $M = \ln B \lg e$, $m = b/a$. The computed curve (using equation 5.) of the depth-profile of the yearly mean soil moisture – together with the observed empirical values – can be seen in Fig. 4. Except for the starting layer matching is surprisingly good. Thus the reality of the connection between the yearly mean soil water stocks and the yearly average amplitude can not be doubted.

In the yearly variation the absolute stocks of the complete layer are subjected to variations; they are decreasing within the drying out period and increasing during the moistening interval. In the course of these

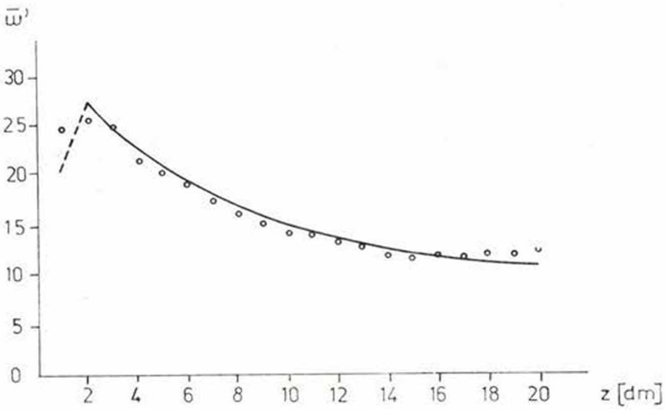

Fig. 4. The depth-profile function of the yearly mean soil moisture together with the empirical values. Erdőhát, 1951–70.

processes also the relative water stocks of the individual soil layers must change, because in both processes the upper surface is the active one, respectively the layers nearest to the surface are the most active.

The curve of the depth moisture profile becomes "straighter" during the drying out period, while in the moistening interval it becomes more "curved". This process has also its yearly variation and extreme positions. From Table 2. we can compute the relative water stock of the main soil layers for every month, but we will show here only the extreme positions (Table 3.). The following obvious statements can be made:

Table 3.

Relative water stocks of different soil layers (%)
Erdőhát, 1951 – 70

Layer (cm)	$\overline{w}'_{\max II}$	$\overline{w}'_{\min IX}$	$\overline{w}'_{VII}$	$\overline{w}'_{XII}$	W'_k	H'_p
0 – 50	37.1	33.9	33.3	38.6	33.7	38.2
50 – 100	24.8	25.8	25.4	25.5	27.2	30.5
100 – 150	19.4	20.5	21.0	18.4	20.3	19.1
150 – 200	18.7	19.8	20.3	17.5	18.8	12.2

In the whole layer the distribution of the water stock is the most balanced in July and the most irregular in December. In July we find one third part of the whole water stock in the highest of the four main layers (0 – 50 cm), which is much more than its proper proportion; in the layer between 50 and 100 cm we get the proper ratio of 1/4, while the two lower layers each have 1/5 -th part, i. e. less than their proper share would be. Similarly, in December the upper layer contains nearly 40% of the whole water stock, in the layer between 50 and 100 cm we can find also about the proper part, while the lower layers have much less than 1/5 th part of the stock. This extreme positions of the profile do not coincide in time with the extreme values of the water stock of the soil, but they develop in the middle of the drying out, respectively of the moistening period. The moisture profiles of yearly maxima and minima of the water stock of the soil (in February respectively September) are different one from another (no parallel shifting), neither do they agree with the profiles of the extreme positions.

The extreme profiles can be most practically compared one with another either using the computed moisture gradients, ore by a linear transformation of the profile curves. This latter method can be seen in Fig. 5. It is obvious that in Fig. 5. the homogeneous moisture profile would be represented by a straight line. The directional tangents of the July and December straight lines, respectively their difference could be a measure on the one hand for the deviation of the extreme profiles from the homogeneous one, and on the other hand they would indicate the limits, within which the actual variations of the moisture profile would take place during the year. One can see in Fig. 5. that the "curvedness" of the moisture profile (its deviation from the homogeneous one) is during the whole year at least one and a half times or twice as big as the possible yearly change of the profile. Further, we can see that the W_k straight line intersects the straight line of the December profile from the left, that of the July profile from right, i. e. its tangent falls between those of these lines. That means that the real moisture profile agrees at least at two dates of the year with the most important hydrophysical moisture profile (i. e. it is parallel to the W_k line) independently of the circumstance that the average absolute water stock of the whole layer is always less than the free ground water capacity. We have to observe also that the more quick variations of the moisture profile take place in the second half of the year (from July to December 5/12 th part), while the more slow ones can be find during the first half of the year (from December to July, 7/12 th part.)

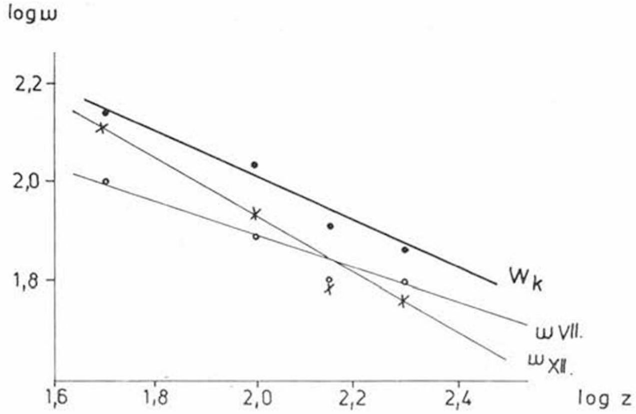

Fig. 5. Average yearly oscillation of the moisture profile at depth, Erdőhát, 1951 – 70.

The results of the analysis of the moisture profile depend also on the thickness of the soil layer considered. If thinner soil layers are under examination, the relative variations of the soil water stock are found more and more greater and also the moisture profiles are becoming more and more liable to changes. It is sufficient to point out that in the upper layers of 0 – 10 cm and 0 – 20 cm in the drying out period we can find a homogeneous moisture profile, resp. an inverse layering, and what is more, this is the characteristic in summer and in late summer.

2. Extreme variations of the soil water stock

The extreme variations of the soil water stock represent at least as important features of the water household of the soil as the average changes. When analysing the extreme soil water stock variations we take into account the following basic material. From the 20 years series of Erdőhát we have arranged the monthly means of the layers in an order of size. The two highest, respectively the two lowest values have been averaged, these represent the extreme values of the soil moisture contet (w_{max}, w_{min}). By these extreme values we may represent the possible yearly oscillation of the water stock. Of course, we can investigate the yearly variation as well as its depth distribution. These variables are therefore empirical functions with two independent variables of the form $w_{max}(z,t)$, $w_{min}(z,t)$, where the depth, $z = 1, 2, \ldots, 200$ cm and the time $t = 1, 2, \ldots, 12$ in months. The depth functions are in most cases cumulative functions (cumultative curves) without any special signs, but the original functions are always denoted with a mark.

In what follows we will investigate in detail the characteristics of the functions $w_{max}(z,t)$ and $w_{min}(z,t)$ and we will describe the most important characteristics which can be derived with the help of the extremity functions. The scheme to be seen on Fig. 6. explains the definition of the new functions to be investigated. We need some special conceptual definitions. Let D be defined as: $D = D_I - D_{II} = d_1 + d_2$, where D_I is the absolute yearly oscillation of the water stock of the soil. It is obvious that we can have $D_{II} \lesseqgtr 0$, as well as $d_1 \lesseqgtr d_3$, respectively, corresponding to the former inequality. Characteristics d_1 and d_2 denote the yearly amplitudes of the extremity functions. Concerning the ratio of d_1 and d_2 we can not draw any conclusion in advance. The variables d_3 and d_4 are called the "wideness characteristics" of the extremity functions. The function $\delta(z,t) = w_{max}(z,t) - w_{min}(z,t)$ can not be defined unambiguously by means of the d(z) functions. What is cetrain is merely that during the year $\delta(t)$ assumes at least once the value of d_3, respectively d_4 in every layer. The yearly amplitude of the $\delta(z,t)$ function is a new depth function, $\Delta(z) = \delta(z,t_{max}) - \delta(z,t_{min})$, defined as "distortion index". In

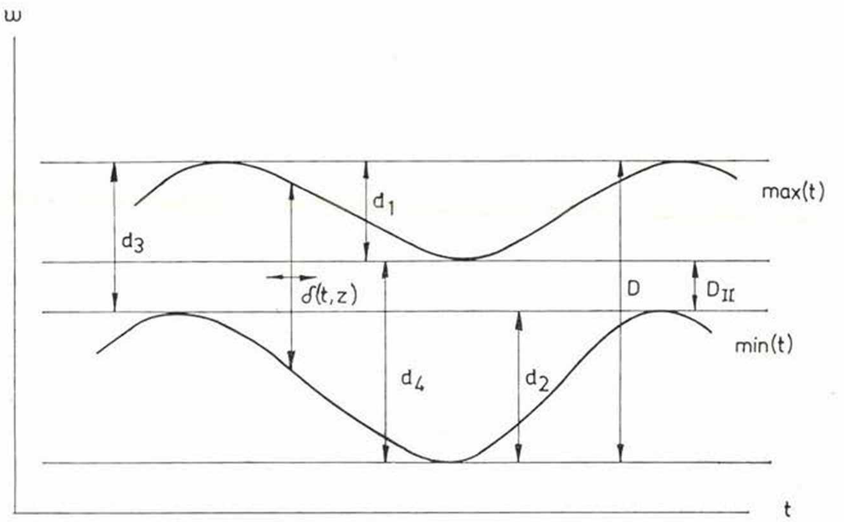

Fig. 6. Model of the empirical extremity functions and characteristics

the course of the detailed analysis we will have to introduce also some further new characteristics at a later stage.

For the $w_{\min}(t)$ function a special interpretation is needed. As a matter of fact, this function represents that part of the water stock, which at the actual climatic conditions does not take part in the active water traffic of the soil. Let us call this water stock in what follows the passive water stock of the bare ground. The expression $w_{\min}$ $(t_{\min})$ is the climatically determined minimal water stock, which never takes part in the water traffic of the bare soil. By using an approximate analogy let us call in what follows the water stock expressed by $w_{\min}(t_{\min})$ the fossil water stock of the bare soil. It is obvious that in case of the presence of a plant cover a part of the passive, respectively fossil water stock could be "activized" by the presence of the plant cover in the layers of the main root zone.

Let us consider at first the connection between the extreme water stocks and the hydrophysical constants. A few observations of general characters are as follows. All the changes of the extreme water stocks remain always between the values of the water stocks of the maximum capillary water capacity and that of the wilting point, (except for $w_{\min}$ at the depth of $0-10$ cm, which is touching, respectively surpassing the limit of Hp in July–September). In the whole year we have: $\mathrm{Hp} < w_{\min} < w_k$, while we may have $w_{\max} \lesseqgtr W_k$. For this later inequality we could have three types in time: 1., $w_{\max} < W_k$ from November to March); 2., $w_{\max} < W_k$ from July to October and 3., $w_{\max} \lesseqgtr W_k$ from April to June, i. e. the cumultative curve of $w_{\max}(z)$ intersect at some depth z_i the cumultative curve of $W_k(z)$. These intersection points can be found from April to June at the depth of 80, 100 and 120 cm one after another.

We can state also a score of such features in the connections with hydrophysical parameters and the $w_{\max}$ (z,t), respectively $w_{\min}$ (z,t) functions which do show more dissimilarity than similarity. Let us present here a few such features.

The profile changes of $w_{\max}(z)$ (during the year) are showing regularities which are very similar to that of the average moisture profile. In the yearly variation the profile changes have a "straightening" and a "curving" section and the relative positions of these to the $W_k(z)$ profile are always well determined. The profiles of the extreme values of $w_{\max}$ (z,t) (March and October) are not identical with the extreme positions of the yearly profile changes. The extreme positions of the profile changes (in agreement with the profile changes of the mean moisture) fall to July, respectively to December. Fig. 7. shows the extremity profile together with the profile of water capacity. The water capacity profile is intersected by the two extremity profiles sumewhere within the whole layer under investigation, namely the extremity profile for July from below and the extremity profile for December from above. That means that the $w_{\max}(z)$ profile agrees with the profile the water capacity, twice during the year and in that case also the absolute values of the $w_{\max}(z)$ water stocks agree with the water capacity. The extreme positions of the

$w_{max}(z)$ profiles strongly deviate from those of a hypothetical homogeneous profile. The difference may equal sometimes the interval of the yearly changes of the profiles.

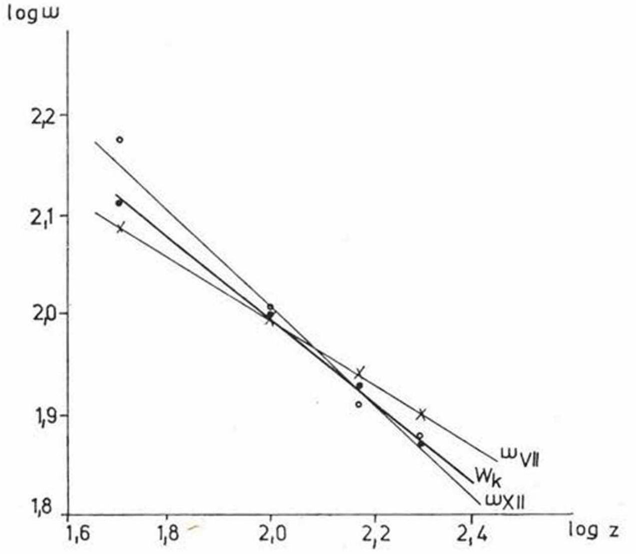

Fig. 7. Yearly oscillation of the depth profile of the maximum water stock. Erdőhát, 1951 – 70.

The $w_{min}(z)$ and Hp profiles show less similarity than the profiles $w_{max}(z)$ and $W_k(z)$. The relative distribution of $w_{min}(z)$ is in general more extreme than that of $w_{max}(z)$, but it is more uniform than that of $Hp(z)$. The $w_{min}(z)$ profiles are always more similar to the corresponding $w_{max}(z)$ profiles, than to the $Hp(z)$ profile. The yearly changes of the $w_{min}(z)$ profile are significant, its extreme positions fall into August, respectively December. The profile of August is about the same as that of July of $w_{max}(z)$ (parallel shifting), while in the December profile the average gradients are much greater than in the December profile of $w_{max}(z)$, but they are still much lower than in the $Hp(z)$ profile.

It is worthwhile to mention the relation of the passive and fossil water stock to the hydrophysical parameters. Part of these water stocks is not available for the plants, but some part of them is contained within the useful water capacity. This part as a relative quantity shall be expressed as follows: $u = \frac{(w_{min} - Hp)}{hw_k}$. The new variable $u(z,t)$ indicates, what part of the useful water capacity turns into passive water and when. It is obvious that the $u(z, t_{min})$ function gives always the corresponding part of the fossil moisture.

The features of the $u(z,t)$ function will not dealt with here in detail, only the depth distribution of the utilizable fossil waterstocks are shown in Fig. 8. We can see a monotonously increasing function with a non-linear upper section because the effect of weather being strong here, but beginning with a certain depth it seems to become rigorously linear. Thus, in this depth zone the total of the fossil water stock above Hp increases by about the same constant value for every layer.

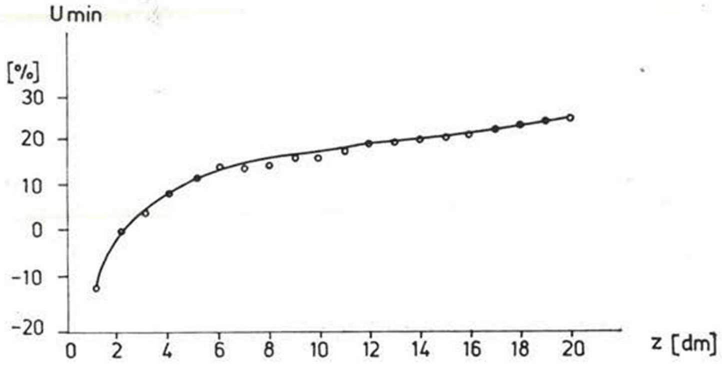

Fig. 8. Depth distribution of the utilizable fossil water stock. Erdőhát, 1951–70.

In the foregoing we defined several extremity characteristics which do not refer to time changes and they depend only on the depth. (Fig. 6.), e. g. D, D_{I}, D_{II}, Δ, d_1, d_2, d_3, d_4. In the course of detailed investigations one can find many interesting features, respectively regularity in the behaviour of these characteristics, and some of them will be shown here.

First of all we consider the domain of the whole water stock soil, of the expressed by the function $w_{\max}$ $(z, t_{\max})$. According to the definition the absolute yearly oscillation of the water stock of the soil is

$$D_{\mathrm{I}} = w_{\max}(t_{\max}) - w_{\min}(t_{\min}) ,$$

and

$$\frac{D_{\mathrm{I}}}{w_{\min}(t_{\min})} = \frac{w_{\max}(t_{\max}) - w_{\min}(t_{\min})}{w_{\min}(t_{\min})} . \tag{7}$$

The expression (7) gives in reality the relative difference of extreme values of the extremity functions, which can characterize the internal structure of the whole domain of the water stock of the soil. The whole domain of the water stock of the soil can be assumed to consist of two partial domains. One of them is the domain of the fossil water stock $w_{\min}(t_{\min})$, which do not change with time, the other is the absolute yearly oscillation (D_{I}) domain, within which all the time changes occur. Both of them are of course functions of the depth.

Fig. 9. shows the depth function of the ratio of the two functions. The ratio decreases very quickly and non-linearly down to about 70–80 cm, then the curve becomes linear and what is more, the value of the

ratio becomes practically constant, i. e. does not depend on depth at all. The value of the ratio is in the whole layer always greater than the unity, i. e. the oscillation domain is always greater than that of constant moisture. In the uppermost layers the ratio turns to assume extreme values, it can surpass several times the unity, i. e. the extreme moisture oscillation extends to the most part of the whole domain. For sake of comparison we give the absolut values of the ratio components for the whole layer:

$$D_{\mathrm{I}}(200) = 276{,}5 \text{ mm}, \qquad w_{\min}(200, t_{\min}) = 191{,}1 \text{ mm}.$$

The variables d_1, d_2, are the yearly amplitudes of the maximum, respectively the minimum functions, while d_3 and d_4 represent the "wideness" characteristics of the extremity functions. Let us consider the depth functions of their ratios.

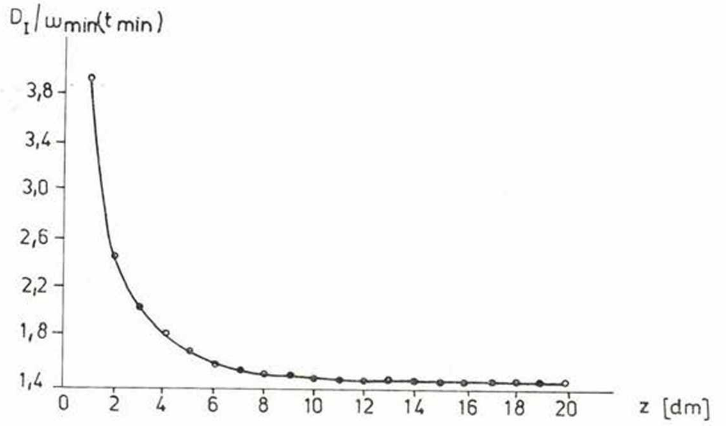

Fig. 9. Depth variation of the structure of the total water stock. Erdőhát, 1951 – 70.

When comparing the behaviour of the amplitudes and wideness characteristics against depth we find a very close similarity, so that itis sufficient to investigate only the depth function of the amplitude ratios. The depth function of the ratio d_1/d_2 presents an interesting regularity (Fig. 10.) at first it is decreasing down to abot 60 – 70 cm, then it is increasing troughout the whole layer, its value is always smaller than the unity and in the whole layer (0 – 200 cm) it is the same as in the uppermost layer (0 – 10 cm). The shape of the ratio-curve is surprisingly similar to that representing the relative water saturation of the soil.

In the relatively more saturated layer (down to 70 cm) the ratio-curve is falling, i. e. $d_2(z)$ is increasing more rapidly relatively, than $d_1(z)$, and what is more we can have $d_1' < d_2'$ also in the praxis. In the relatively less saturated layer (below 70 cm) the ratio-curve is rising, i. e. $d_1(z)$ is increasing more rapidly than $d_2(z)$, and what is more, there can also be found that $d_1' > d_2'$ in the praxis. This indicates that the behaviour of the two extremity functions is in close interrelation with the relative moisture state of the

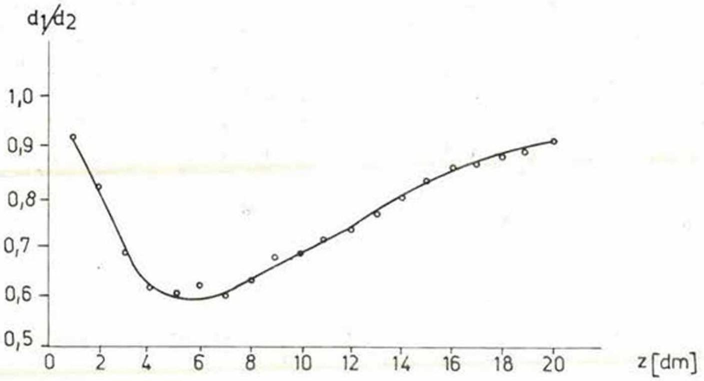

Fig. 10. Empirical depth function of the ratio of extreme amplitudes. Erdőhát, 1951 – 70.

soil and it is inversely reacting to it. Both amplitudes $d_1'(z)$ and d_2' are decreasing, but as long as $d_1'(z)$ changes relatively slowly and to a little extent, $d_2'(z)$ changes within the whole layer by an order of magnitude. Thus, for the relative moisture state of the soil it is the minimum function which is most indicative.

The depth functions of the amplitudes d_1 and d_2 could hardly be determined separately in a mathematical form. We have determined the depth function of the differences of the two amplitudes in a purely empirical manner as follows:

$$\frac{d_2 - d_1}{d_2} = a\, z^b\, e^{cz}\,, \tag{8}$$

from which we get

$$d_1 = d_2(1 - a\, z^b\, e^{cz})\,, \tag{9}$$

where a, b and c are empirical constants. The agreement between computed and empirical values is sufficiently good, so that an amplitude can be calculated approximately from the other one when necessary. On the other hand, the sum of the two extreme amplitudes

$$D - d_1 + d_2$$

represents also a depth function in the same manner as the mean amplitude. The function of the $D(z)$ characteristic analogue to (1) is (Fig. 2.):

$$D(z) = D_1 \frac{B^{\lg z}}{z}\,. \tag{11}$$

In (11) D_1 is the value of the characteristic in the starting layer, B is a general constant and z is a dimensionless depth variable.

The relative difference of the empirical values and of those computed from (11) is less than 4% (it surpasses this limit only at 20 and 30 cm)

It is proper to suppose that the equation (11) represents an exact function such as that of the average amplitude. From equations (10) and (11) we have to conclude that the extermity amplitudes $d_1(z)$ and $d_2(z)$ despite of their apparently deviating behaviour are very closely interrelated one with another and they are conditional upon one another.

Differentiating equation (11) we obtain the original depth function

$$D'(z) = B_1\, D_1 \frac{B^{\lg z}}{z^2}. \tag{12}$$

The curve of the function (12) (together with the empirical values) appears in Fig. 11. We can easily see that the somewhat poor matching in a few cases can only be due to the random errors of the empirical values.

The position of the two extremity functions against each other – represented in some part by the wideness characteristics – is also typically changing with depth. The depth curve of the ratio d_3/d_4 is so similar to the curve of the Fig. 10. that it is not worth showing separately. The depth function of the relative differences of d_3 and d_4 is the same as (8), respectively (9), only the values of the parameter are different. The depth function of the sum of the wideness characteristics (d_3+d_4) can not be constructed in an analogous manner to the equation (12).

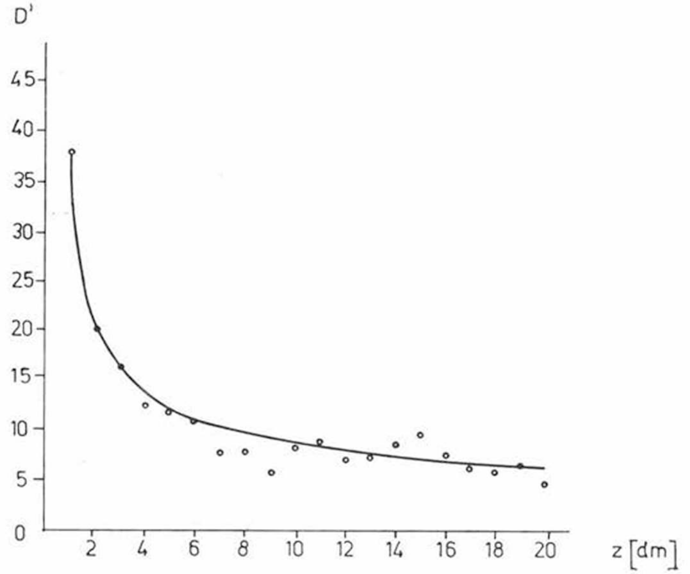

Fig. 11. Depth function constructed from the extreme amplitude sums. Erdőhát, 1951 – 70.

The depth function of the ratio Δ/D_{I} is shown in Fig. 12. Out of the characteristics given above D_{I} is the greatest, Δ the smallest (in absolute value). The function $\Delta/(z)$ can be interpreted as a certain "distortion" index. One can imagine that $\Delta(z)$ figures in the absolute yearly oscillation as an additive term. If the distortion would be eliminated, i. e. the curve of any extremity function could be derived by a simple parallel shifting from the another, then the absolute yearly oscillation would diminish by $\Delta(z)$ or by a proportional part of it. Thus, the ratio Δ/D_{I} indicates the share of this distortion in the yearly oscillation.

The curve (Fig. 12.) is about the reflected image of the curves of ratios d_1/d_2, respectively d_3/d_4. Both component parts of the ratio are increasing, but in the upper layers $\Delta(z)$ is increasing steeper than $D_{\mathrm{I}}(z)$, while it becomes reversed in the lower layers. Though $\Delta'(z)$ in the $0-10$ cm layer assumes its highest value and it diminishes downwards, the relative distortion index $[\Delta(D_{\mathrm{I}})]$ is showing its maximum in the $0-60$ cm layer (it surpasses 1/3) and it decreases quickly with the depth, and its value for the whole layer is only 17%. Thus, the distortion index is also connected to the relative (water) saturation state of the soil.

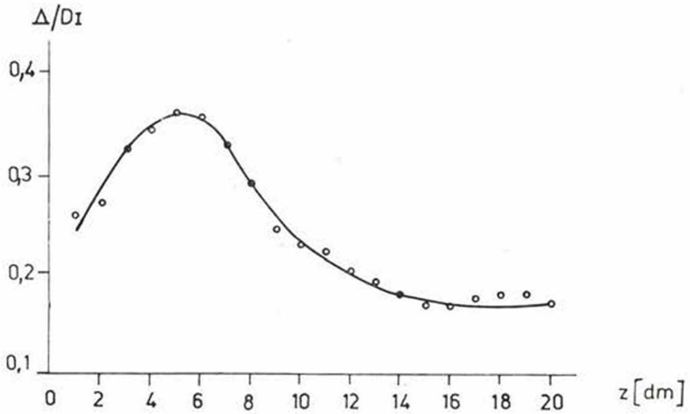

Fig. 12. Depth distribution of the relative distortion index. Erdőhát, 1951 – 70.

Finally we will introduce the function $p(z,t) = \delta(z,t) \ w(z,t)$ showing (in the seasonal variation) the ratio of the extrem eoscillation and of the average water stock. In the yearly variation of $p(z)$ we can separate three types, as shown on Fig. 13. Two of the types correspond to the extreme positions of the yearly variation and the third one is the dominating one. The first type (January curve) is characterised by the circumstance that in the upper layers (down to $60-70$) cm $p(z)$ remains practically constant, then it is rising. With the second end of the summer type (August curve) the picture is principially reversed: the upper section is (not linearly) decreasing and the curve becomes constant in the lower section. The third dominant type (May curve) is a combination of the two extremity types: in the upper section it is showing the summer type, while in the lower one it is like the winter type. In the major part of the year (in seven months altogether) the third type prevails.

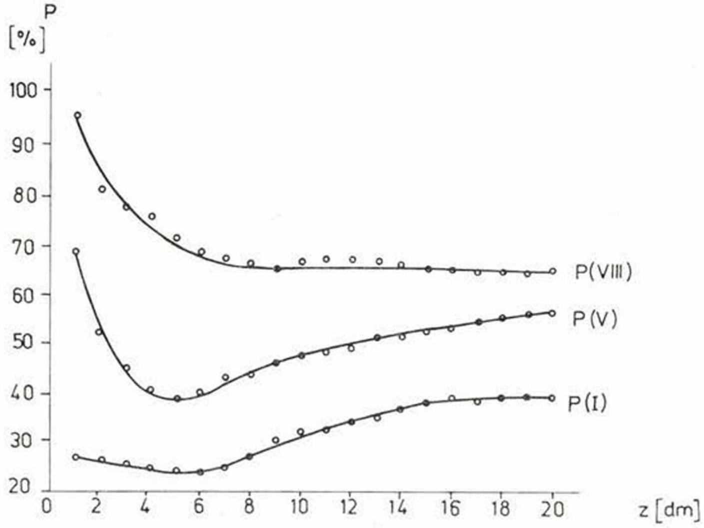

Fig. 13. Three basic types of the empirical $p(z,t)$ function. Erdőhát, 1951 – 70.

This is clearly supporting our earlier conclusion that the yearly variability of the soil moisture is closely, but inversely correlated with the relative (water) saturation of the soil. This is truly reflected in the seasonal variation and depth distribution too.

3. Connections of the average and extreme water stocks

Up to now we have investigated the average or extreme water stock changes and looked for regularities which showed themselves valid independently from one another. Our further aim is to detect the connections between mean and extreme variations which can indicate also the most general regularities of the moisture variations. From the numerous possibilities we will treat in detail here only two relations, the connection between the extremity characteristics and the yearly mean moisture content as well as some connections of the mean yearly amplitude.

We are showing on a common figure (Fig. 14.) the depth curves of the ratios $D/\overline{w}$ and $D_{\mathrm{I}}/\overline{w}$. Both ratios have a common denominator and the numerators are also very close one to another with regards both their physical content and absolute values, as well as their depth variations. The features of the ratio-depth functions partly agree, partly deviate. Both functions surpass the unity in the uppermost section (down to about 30 cm), while in the other layers their value is less than 1. That means that the absolute yearly amplitude of the soil water stock may be greater than the average yearly stock. And that is true so much the more for the D-characteristic, since in the uppermost layers we have $D > D_{\mathrm{I}}$. At the depth of 40 cm the two curves intersect, i. e. here we must have $D = D_{\mathrm{I}}$ and $D_{\mathrm{II}} = 0$. The depth curves of both

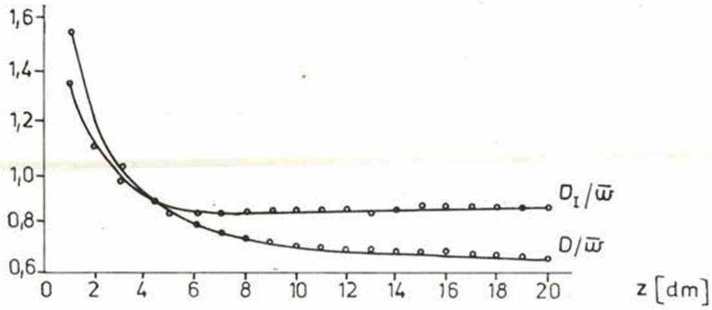

Fig. 14. Depth profiles of important characteristic ratios. Erdőhát, 1951 – 70.

ratios represent a non-linearly decreasing function. The ratio $D/\bar{w}$ is a monotonously decreasing function in the whole layer, while the curve of $D_I/\bar{w}$ is much more complicated. This one is also decreasing down to 70 cm (it has a local minimum hardly distinguishable there), then it increases somewhat, after it remains strictly constant in the layers between 100 and 200 cm. The similarity with the corresponding $p(z)$ functions is apparent (Fig. 13.).

Characteristic connections can be established also by using the mean yearly amplitude. An example for this can be seen on Fig. 15. The connection between mean and absolute yearly amplitude is surprisingly close and the ratio depth function is strictly linear. The ratio

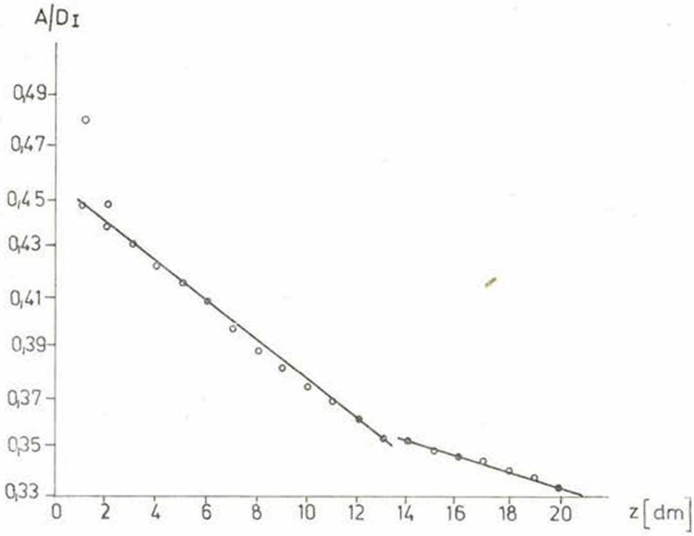

Fig. 15. Empirical depth functions of the ratio of the mean amplitude and absolute yearly oscillation. Erdőhát, 1951 – 70.

values do not deviate from the adjusted straight line except in the uppermost layer between $0-20$ cm and a striking deviation is shown only by the $0-10$ cm layer. In the whole layer two straight lines can be adjusted to the empirical values. The first one is valid for the $0-130$ cm layer, the second one for the $130-200$ cm one. The physical cause respectively the exact explanation of the deviation of the two domains is still lacking.

The ratio-depth-function of A/D_{I} can be described by a linear regression equation as follows:

$$\varphi(z) = \beta - \alpha z_z \,. \tag{13}$$

since for the two domains two equations are valid, the values of the corresponding empirical parameters are different too. From the equations (1) and (13) we are able to express the depth function of the absolute yearly amplitude total (Fig. 2.) as follows:

$$D_{\mathrm{I}}(z) = A_1 \frac{B^{\lg z}}{\beta z - \alpha z^2} \,. \tag{14}$$

The matching of the empirical and computed D_I values, obtained from equation (14) is very good. The relative differences fall within $0-3\%$ and, in spite of the linear interpolation, even in the layer between 0 and 10 cm does not surpass $6{,}8\%$, which is still aceptable.

Differentiating equation (14) we get the original depth function of the absolute yearly oscillation

$$D_{\mathrm{I}}'(z) = A_1 \frac{B^{\lg z}}{\beta z - \alpha z^2} \left(\frac{B_1}{z} + \frac{1}{p-z} \right), \tag{15}$$

where $p = \beta/\alpha$. We have to mention that (15) can be written by using equations (1) and (2) in the following form:

$$D_{\mathrm{I}}'(z) = A(z) \frac{\alpha}{(\beta - \alpha z)^2} + A'(z) \frac{1}{(\beta - \alpha z)} \,. \tag{16}$$

Finally we will show the connection between the average yearly amplitude and the amplitudes of the extremity-functions (Fig. 16.). By this we get an additional proof of the fact that certain features of the two extremity-functions are significantly different. It is obvious that the yearly distribution function of the average water stock must be located between the extremity-functions, but the average amplitude does not necessarily fall between the extreme amplitudes. In reality we have $d_1(z) \leqslant A(z) < d_2(z)$, as it can be checked by means of the values of the ratios. The ratio A/d_2 is a function of the same type as d_1/d_2 (Fig. 10.), while A/d_1 is a reflexted image of the previous one. Thus it follows that the depth function of the mean amplitude concerning its absolut values and other features (relative increments etc.) is very close to the amplitude of the maximum function and at the same time it differs significantly from the amplitude of the minimum function.

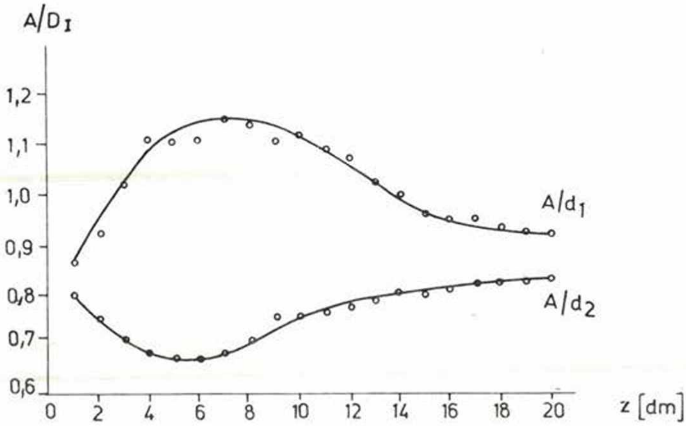

Fig. 16. Depth function curve of the absolute yearly oscillation. Erdőhát, 1951 – 70.

By the analyses above we have demonstrated though not exhausted the rich content of this domain of problems or its possibilities. A more detailed meteorological analysis of the connections and regularities detected here as well as their interpretation will be the subject of further studies.

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